Misc.

Q 8 → GP

$a = 5$
$r = 2$

GP
5, 10, 20, 40, ...

$S_n = 315$

$\frac{a(r^n - 1)}{(r-1)} = 315$

$\frac{5(2^n - 1)}{(2-1)} = 315$

$2^n - 1 = 63$

$2^n = 64$

$2^n = 2^6$

$\boxed{n = 6}$

last term

$a_n = ar^{n-1}$

$= 5(2)^{6-1}$

$= 5 \times 32$

$\boxed{a_n = 160}$

Q9 → GP
$a = 1$

$a_3 + a_5 = 90$

$ar^2 + ar^4 = 90$

$r^2 + r^4 = 90$ $(a = 1)$

Lets say $r^2 = x$

$x + x^2 = 90$

$x^2 + x - 90 = 0$

$x^2 + 10x - 9x - 90 = 0$

$x(x+10) - 9(x+10) = 0$

$(x-9)(x+10) = 0$

$x = 9$ or -10

$r^2 = 9$ / $r^2 = -10$ X (neglect)

$\boxed{r = \pm 3}$

Q10 →   Let 3 terms be ....

GP → a, ar, ar^2

$$a + ar + ar^2 = 56 \quad \text{—(1)}$$

$$a(1+r+r^2) = 56$$

New terms ⇒ AP

$$a-1,\ ar-7,\ ar^2-21$$

$$\boxed{b = \frac{a+c}{2}}$$

$$ar - 7 = \frac{(a-1)+(ar^2-21)}{2}$$

$$2(ar-7) = a + ar^2 - 22$$

from (1)
$a + ar^2 = 56 - ar$

$$2(ar-7) = (56-ar) - 22$$

$$2ar - 14 = 34 - ar$$

$$3ar = 48 \Rightarrow \boxed{ar = 16} \quad \text{—(2)}$$

from (1)

$$a(1+r+r^2) = 56$$

from (2)

$$\frac{\cancel{16}^{\,2}}{r}(1+r+r^2) = \cancel{56}\ 7$$

$$2 + 2r + 2r^2 = 7r$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2 \text{ or } \frac{1}{2}$$

If $r = 2$

from (2) $a = 8$ ⇒

a, ar, ar^2

8, 16, 32

If $r = \frac{1}{2}$

$a = 32$ ⇒

32, 16, 8

Q11 →   GP consists of even number of terms ...

~~$a, ar, \ldots\ldots, ar^{n-1}$~~

$$a_1, a_2, a_3, a_4, \ldots\ldots a_{n-1}, a_n$$

total terms = n

n is even

$a, ar, ar^2, ar^3, \ldots, ar^{n-2}, ar^{n-1}$

Sum of all terms

$S_n = \frac{a(r^n - 1)}{(r-1)}$

Terms at odd places

$a, ar^2, ar^4, \ldots, ar^{n-2}$ (∵ n is even)

$A = a, \; R = r^2$

$N = ??$

$A_n = AR^{N-1}$

$\cancel{a}r^{n-2} = \cancel{a}(r^2)^{N-1}$

$n - \cancel{2} = 2N - \cancel{2}$

$\underline{N = \frac{n}{2}}$

$S_N = \frac{A(R^N - 1)}{(R-1)}$

$S_N = \frac{a((r^2)^{\frac{n}{2}} - 1)}{(r^2 - 1)}$

$S_N = \frac{a(r^n - 1)}{(r^2 - 1)}$

Sum of all the terms is 5 times the sum of terms occupying odd places

$S_n = 5 S_N$

$\cancel{\frac{a(r^n-1)}{(r-1)}} = 5 \cancel{\frac{a(r^n-1)}{(r^2-1)}}$

$(r^2 - 1) = 5(r-1)$

$(r+1)\cancel{(r-1)} = 5\cancel{(r-1)}$ Since $r \neq 1$

$r + 1 = 5$

$\boxed{r = 4}$

Check...

$r = 4$

3, 12, 48, 192

$S_2 = 3 + 12 = 15$

$S_{odd} = 3 \times 5 = 15$

Q12 → AP

$a = 11$

$d = ??$

$a_n = a + (n-1)d$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$11, 11+d, 11+2d, 11+3d, \ldots, 11+(n-1)d$

Sum of First 4 terms $= \frac{4}{2}(2(11) + (4-1)d) = 56$

$\cancel{2}(22 + 3d) = \cancel{56}\ 28$

$3d = 6$

$\boxed{d = 2}$

for last 4 terms lets consider AP in reverse

First $\cancel{A} =\ a_n = a + (n-1)d$
$= 11 + (n-1)2$

Total terms in the series
$n = ??$

$D = -2$

$N = 4$

$$S_N = \frac{N}{2}\left(2A + (N-1)D\right)$$

$$S_4 = \frac{4}{2}\left(2\left(11 + (n-1)2\right) + (4-1)(-2)\right)$$

$$\overset{56}{\cancel{112}} = \cancel{2}\left(22 + (n-1)4 - 6\right)$$

$$56 = 16 + 4n - 4$$

$$56 = 4n + 12$$

$$44 = 4n$$

$$\boxed{n = 11}$$

Q13 →

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

$\frac{A}{B} = \frac{C}{D} \Rightarrow$ C&D $\frac{A+B}{A-B} = \frac{C+D}{C-D}$

Show a, b, c are in GP

C&D

$$\frac{a}{bx} = \frac{b}{cx}$$

$$\frac{a}{b} = \frac{b}{c} \quad (1)$$

CD

$$\frac{(b+\cancel{cx}) + (b-\cancel{cx})}{(\cancel{b}+cx) - (\cancel{b}-cx)} = \frac{(c+\cancel{dx}) + (c-\cancel{dx})}{(\cancel{c}+dx) - (\cancel{c}-dx)}$$

$$\frac{\cancel{2}b}{\cancel{2}cx} = \frac{\cancel{2}c}{\cancel{2}dx}$$

$$\frac{b}{c} = \frac{c}{d} \quad (2)$$

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$$

— x — x — x —

a, b, c & d are in GP

Q14 →

GP

Let S be the sum of n terms

& P be the product of n terms

& R be the sum of reciprocals of n terms

Show $P^2 R^n = S^n$

$S = a + ar + \cdots + ar^{n-1}$

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

$P = (a)(ar)(ar^2) \cdots\cdots (ar^{n-1})$

$P = (\underbrace{a.a.a \cdots a}_{n \text{ times}})(\underbrace{r^1.r^2.r^3 + \cdots r^{n-1}}_{n-1 \text{ terms}})$

$\left| \sum_{r=1}^{n} r = \frac{n(n+1)}{2} \right|$

$P = a^n \, r^{1+2+\cdots+n-1}$

$$\boxed{P = a^n \, r^{\frac{n(n-1)}{2}}}$$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \cdots\cdots + \frac{1}{ar^{n-1}}$ (GP)

$A = \frac{1}{a}$ $R = \frac{1}{r}$ $N = n$

~~SA~~

$$S_N = \frac{A(R^N - 1)}{(R - 1)}$$

$$R = \frac{\frac{1}{a}\left(\frac{1}{r^n} - 1\right)}{\left(\frac{1}{r} - 1\right)} \Rightarrow \frac{1}{a} \frac{\left(\frac{1 - r^n}{r^n}\right)}{\left(\frac{1 - r}{r}\right)}$$

$$R = \frac{1}{a} \frac{1-r^n}{r^n} \times \frac{r}{1-r}$$

$$\boxed{R = \frac{1}{a r^{n-1}} \frac{(r^n - 1)}{(r-1)}}$$

Prove $P^2 R^n = S^n$

$$LHS = P^2 R^n$$

$$\left(a^n r^{\frac{n(n-1)}{2}}\right)^2 \left(\frac{1}{a r^{n-1}} \times \frac{r^n - 1}{r-1}\right)^n$$

$$\Rightarrow a^{2n} \cancel{r^{n(n-1)}} \frac{1}{a^n \cancel{r^{n(n-1)}}} \left(\frac{r^n - 1}{r-1}\right)^n$$

$$= a^n \left(\frac{r^n - 1}{r-1}\right)^n$$

$$= \left[a \left(\frac{r^n - 1}{r-1}\right)\right]^n$$

$$= S^n$$

$= RHS$ hence proved...

Q15 → AP ⇒ A, D

$a_p = a \Rightarrow A + (p-1)D = a$ — (1)

$a_q = b \Rightarrow A + (q-1)D = b$ — (2)

$a_r = c \Rightarrow A + (r-1)D = c$ — (3)

Show

$$(q-r)a + (r-p)b + (p-q)c = 0$$

$LHS = (q-r)(A+(p-1)D) + (r-p)(A+(q-1)D) + (p-q)(A+(r-1)D)$

$(q-r)A + (q-r)(p-1)D + (r-p)A + (r-p)(q-1)D + (p-q)(A) + (p-q)(r-1)D$

$A(\cancel{q}\cancel{-r}+\cancel{r}\cancel{-p}+\cancel{p}\cancel{-q}) + D(\cancel{pq}-q-\cancel{rp}+r+\cancel{rq}-r-\cancel{pq}+p + \cancel{pr}-p-\cancel{qr}+q)$

$= A(0) + D(0)$

$= 0 = RHS$ Hence proved...

Q16. $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP

If A, B, C are in AP $2B = A+C$

Prove a, b, c are in AP

$2b = a+c$

$2b\left(\frac{1}{c}+\frac{1}{a}\right) = a\left(\frac{1}{b}+\frac{1}{c}\right) + c\left(\frac{1}{a}+\frac{1}{b}\right)$

$2b\left(\frac{a+c}{ac}\right) = a\left(\frac{b+c}{bc}\right) + c\left(\frac{b+a}{ab}\right)$

$2\left(\frac{ab+bc}{ac}\right) = \frac{ab+ac}{bc} + \frac{bc+ac}{ab}$

$\frac{2(ab+bc)}{\cancel{ac}} = \frac{a^2b+a^2c+bc^2+ac^2}{ab\cancel{c}}$

$2(ab^2+b^2c) = a^2b+a^2c+bc^2+ac^2$

$\cancel{2b^2(a+c)}$

$\cancel{2ab^2-a^2b+2b^2c-bc^2 = a^2c+ac^2}$

$\cancel{ab(2b-a)+bc(2b-c) = ac(a+c)}$

~~$2B = A+C$~~

$B-A = C-B = d$

$B-A=C-B$

$$ab^2+b^2c-a^2b-a^2c = bc^2+ac^2-ab^2-b^2c$$

$$ab(b-a)+c(b^2-a^2) = bc(c-b)+a(c^2-b^2)$$

$$(b-a)\{ab+c(b+a)\} = (c-b)(bc+a(c+b))$$

$$(b-a)\{\cancel{ab+bc+ac}\} = (c-b)\{\cancel{ab+bc+ac}\}$$

$$b-a = c-b$$

$$2b=a+c \Rightarrow a, b, c \text{ are in AP}$$

— x — x — x —

Alternatively...

$$a\left(\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{c}+\frac{1}{a}\right),\ c\left(\frac{1}{a}+\frac{1}{b}\right) \text{ are in AP}$$

Transformations →

En $\cancel{A}\cancel{P}$

AP 2, 4, 6 ...

$\downarrow +1$

AP 3, 5, 7

$\downarrow \div 2$

AP $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

$\downarrow \times 3$

AP $\frac{9}{2}, \frac{15}{2}, \frac{21}{2}$

$$\frac{ac+ab}{bc}+1,\ \frac{ab+bc}{ac}+1,\ \frac{bc+ac}{ab}+1 \text{ are in AP}$$

$\Downarrow +1$

$$\frac{ab+bc+ac}{bc},\ \frac{ab+bc+ac}{ac},\ \frac{ab+bc+ac}{ab} \text{ are in AP}$$

$\Downarrow \div ab+bc+ac$

$$\frac{1}{bc},\ \frac{1}{ac},\ \frac{1}{ab} \text{ are in AP}$$

$\Downarrow \times abc$

$$a,\ b,\ c \text{ are in AP.}$$

Q17 → a, b, c & d are in GP

a, ar, ar^2, ar^3

$\boxed{A, B, C \text{ are in GP} \Rightarrow B^2 = AC}$

Prove → $(a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in GP

$$\frac{b^n+c^n}{a^n+b^n} = \frac{(ar)^n+(ar^2)^n}{a^n+(ar)^n} = \frac{\cancel{a^n}\, r^n \cancel{(1+r^n)}}{\cancel{a^n}\cancel{(1+r^n)}}$$

$$\Rightarrow \frac{c^n+d^n}{b^n+c^n} = r^n \Rightarrow GP$$

—x—x—x—x—

Q18 ⇒

a & b are roots of ...

$$x^2-3x+p=0$$

$$\boxed{\alpha+\beta = \frac{-b}{a}} \qquad \boxed{\alpha\beta = \frac{c}{a}}$$

$$a+b = -\frac{(-3)}{1} \qquad ab = \frac{p}{1}$$

$$\underline{\overline{a+b=3}} \quad -(1) \qquad \underline{\overline{ab=p}} \quad -(2)$$

⇒ Already done (Last lecture)

Q19 a, b ⇒ Two positive numbers

$$\frac{AM}{GM} = \frac{m}{n}$$

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

C&D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$$

$$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$$

$\Downarrow$ C&D

$$\frac{(\sqrt{a}+\cancel{\sqrt{b}})+(\sqrt{a}-\cancel{\sqrt{b}})}{(\cancel{\sqrt{a}}+\sqrt{b})-(\cancel{\sqrt{a}}-\sqrt{b})} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

⇓ squaring

$$\frac{a}{b} = \frac{m+\cancel{n}+m-\cancel{n}+2\sqrt{m^2-n^2}}{m+\cancel{n}+m-\cancel{n}-2\sqrt{m^2-n^2}}$$

$$\frac{a}{b} = \frac{\cancel{2}(m+\sqrt{m^2-n^2})}{\cancel{2}(m-\sqrt{m^2-n^2})}$$

$$a:b = (m+\sqrt{m^2-n^2}):(m-\sqrt{m^2-n^2})$$

Hence proved.

Q20 →

a, b, c are in AP

$2b = a+c$ — ①

b, c, d are in GP

$c^2 = bd$ — ②

$\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in AP

$\frac{2}{d} = \frac{1}{c}+\frac{1}{e}$ — ③

$$\frac{2}{d} = \frac{e+c}{ce}$$

$$\boxed{d = \frac{2ce}{c+e}}$$

To prove ...

a, c, e are in GP

$c^2 = ae$

from ① $b = \frac{a+c}{2}$ — 1'

from ③ $d = \frac{2ce}{c+e}$ — 2'

Put ①' & ②' in ②

$$c^2 = \frac{a+c}{\cancel{2}} \times \frac{\cancel{2}ce}{c+e}$$

$$c^2 = \frac{ace+c^2e}{(c+e)}$$

$$c^3 + \cancel{c^2e} = ace + \cancel{c^2e}$$

$$c^{\cancel{3}2} = a\cancel{c}e \quad (c \neq 0)$$

$$c^2 = ae$$

a, c & e are in GP
(Hence proved)

Q21 →

$S_n = ??$

(ii) $.6 + 0.66 + 0.666 + \dots$

$\cancel{6}\ \cancel{0.9}$

$$6\left(.1 + 0.11 + 0.111 + \cdots 0.\underbrace{11\ldots1}_{n \text{ times}}\right)$$

$$\frac{6}{9}\left(0.9 + 0.99 + 0.999 + \cdots 0.\underbrace{999\ldots9}_{n\text{times}}\right)$$

$$\frac{2}{3}\left((1-0.1) + (1-0.01) + (1-0.001) + \cdots + (1-0.\underbrace{00\ldots01}_{\substack{n \text{ terms} \\ \text{after decimal}}})\right)$$

$$\frac{2}{3}\left(\underbrace{(1+1+1\cdots+1)}_{n \text{ times}} - \underbrace{(0.1+0.01+0.001 \cdots\cdots)}_{n \text{ terms}}\right)$$

GP

$a = 0.1 = \frac{1}{10}$

$r = 0.1 = \frac{1}{10}$

$S_n = \frac{a(r^n - 1)}{(r-1)}$

$$\frac{2}{3}\left(n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{\left(1-\frac{1}{10}\right)}\right)$$

$$\cancel{\frac{2}{3}\left(n - \frac{1}{10}\frac{\left(\frac{10^n - 1}{10^n}\right)}{\frac{9}{10}}\right) = \frac{2}{3}\left(n - 10^n - 1\right)}$$

$$\frac{2}{3}\left(n - \frac{\frac{1}{10}(1-10^{-n})}{\frac{9}{10}}\right)$$

$$\frac{2}{3}\left(n - \frac{(1-10^{-n})}{9}\right) = \boxed{\frac{2}{3}n - \frac{2}{27}(1-10^{-n})}$$

Q22 →

$$2\times4 + 4\times6 + 6\times8 + \cdots\cdots$$

$S_1 = 2, 4, 6, \cdots \Rightarrow$ AP

$a_n^1 = 2 + (n-1)2$

$\boxed{a_n^1 = 2n}$

$S_2 = 4, 6, 8, \cdots$ AP

$a_n^2 = 4 + (n-1)2$

$\boxed{a_n^2 = 2n+2}$

$a_n = \cancel{0}\, 4 + (n-1)2$

$\boxed{a_n^2 = 2n+2}$

$a_n = a_n^1 \times a_n^2 = (2n)(2n+2)$

$a_n \Rightarrow 4n(n+1)$

$a_n \Rightarrow 4n^2 + 4n$

$S_n = \sum a_r$

$= 4\sum r^2 + 4\sum r$

$= 4\frac{n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2}$

$a_{20} = 4(20)^2 + 4(20)$

$\Rightarrow 1600 + 80$

$= \underline{1680}$

— x — x — x — x —

Q23 →

$S_n = 3 + \overbrace{7 + 13 + 21 + 31 + \cdots\cdots}^{n-1 \text{ terms}} + a_n$

$S_n = \quad 3 + 7 + 13 + 21 \cdots\cdots a_{n-1} + a_n$

$0 = 3 + (\underbrace{4 + 6 + 8 + 10 \cdots\cdots}_{n-1 \text{ terms}}) - a_n$

$a_n = 3 + \frac{n-1}{2}\left(2(4) + (n-1-1)2\right)$

$a_n = 3 + \frac{n-1}{2}(8 + 2n - 4)$

$a_n = 3 + (n-1)(n+2)$

$a_n = 3 + n^2 + n - 2$

$\underline{a_n = n^2 + n + 1}$

— x — x — x —

$S_n = \sum_{r=1}^{n} a_r$

$S_n = \sum_{r=1}^{n} (r^2 + r + 1)$

$S_n = \sum r^2 + \sum r + \sum 1$

$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$

$S_n = n\left(\frac{(n+1)(2n+1)}{6} + \frac{n+1}{2} + 1\right)$

$S_n = n\left(\frac{2n^2 + 3n + 1 + 3n + 3 + 6}{6}\right)$

$$S_n = \frac{n(2n^2+6n+10)}{6}$$

$$S_n = \frac{n(n^2+3n+5)}{3}$$

Q24-

S_1, S_2 & S_3 be sum of first n natural numbers & ~~their~~ their squares & their cubes

$$S_1 = \frac{n(n+1)}{2}$$

$$S_2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = \left(\frac{n(n+1)}{2}\right)^2$$

Show that

$$9S_2^2 = S_3(1+8S_1)$$

$$LHS = 9S_2^2 = 9\left(\frac{n(n+1)(2n+1)}{6}\right)^2$$

$$= \frac{n^2(n+1)^2(2n+1)^2}{4}$$

$$RHS = S_3(1+8S_1)$$

$$\left(\frac{n(n+1)}{2}\right)^2\left(1+\frac{\cancel{8}^{4}\,n(n+1)}{\cancel{2}}\right)$$

$$\frac{n^2(n+1)^2}{4}(1+4n^2+4n)$$

$$\frac{n^2(n+1)^2(2n+1)^2}{4}$$

LHS = RHS

Hence proved..

Q25 → $S_n = \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} - - - - -$

$$a_n = \frac{1^3+2^3+3^3+ \cdots n^3}{1+3+ + \cdots (2n-1)}$$

1, 3, 5, - - - -

$a_n = 1+(n-1)2$

$$a_n = \frac{1+2+3+\dots n}{1+3+ + \dots (2n-1)}$$

$$a_n = 1+(n-1)2$$
$$2n-1$$

$$S_n = \frac{n}{2}(1+2n-1)$$
$$= n^2$$

$$a_n \Rightarrow \frac{\left(\frac{n(n+1)}{2}\right)^2}{n^2}$$

$$a_n = \left(\frac{n+1}{2}\right)^2$$

$$a_n = \frac{n^2+2n+1}{4} \quad = a_n = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

— x — x — x —

$$S_n = \sum_{r=1}^{n} a_r$$

$$S_n = \sum_{r=1}^{n}\left(\frac{r^2}{4} + \frac{r}{2} + \frac{1}{4}\right)$$

$$= \frac{1}{4}\sum r^2 + \frac{1}{2}\sum r + \frac{1}{4}\sum 1$$

$$\Rightarrow \frac{1}{4}\,\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\,\frac{n(n+1)}{2} + \frac{1}{4}n$$

$$\Rightarrow \frac{n}{4}\left(\frac{(n+1)(2n+1)}{6} + (n+1) + 1\right)$$

$$\frac{n}{4}\left(\frac{2n^2+3n+1+6n+12}{6}\right)$$

$$= \frac{n(2n^2+9n+13)}{24}$$

— x — x — x —

Q26 → HW

$$\frac{1\times2^2 + 2\times3^2 + \dots + n(n+1)^2}{1^2\times2 + 2^2\times3 \dots + n^2(n+1)}$$

$$a_n^1 = n(n+1)^2 = n(n^2+2n+1) = n^3+2n^2+n$$

$$S_n^1 = \sum_{r=1}^{n} a_r^1 = \sum_{r=1}^{n} r^3+2r^2+r$$

$\vdots$

(Sum of numerator) $S_n^1 = ??$

$$a_n^2 = n^2(n+1) = n^3+n^2$$

(Denominator) $S_n^2 = \sum_{r=1}^{n} r^3+r^2$

$\vdots$

$S_n^2 = ??$

$$S_n = \frac{S_n^1}{S_n^2} \Rightarrow \frac{3n+5}{3n+1}$$

H/W